

Risk Analysis of a Processing System Considering Multivariate Dependency and Correlation

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OUTLINE



- Research motivation
- Technique used to model process failure
- Experimental verification
- Comparison of models' performances
- Discussions
- References

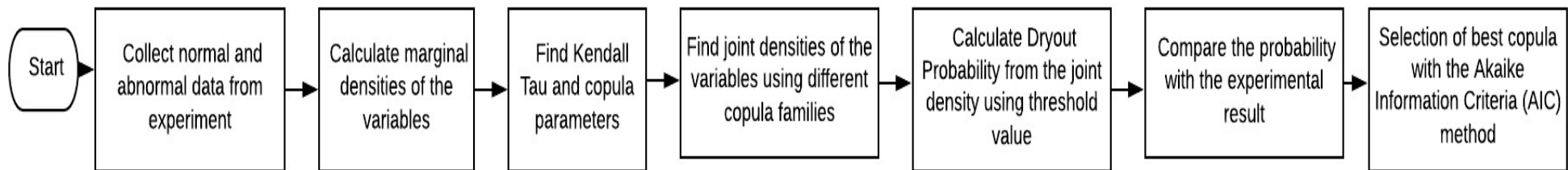
MOTIVATION OF THE RESEARCH

- Inter-connectivity among process units
- Dependency among the process variables
- Use of rank correlation instead of linear correlation

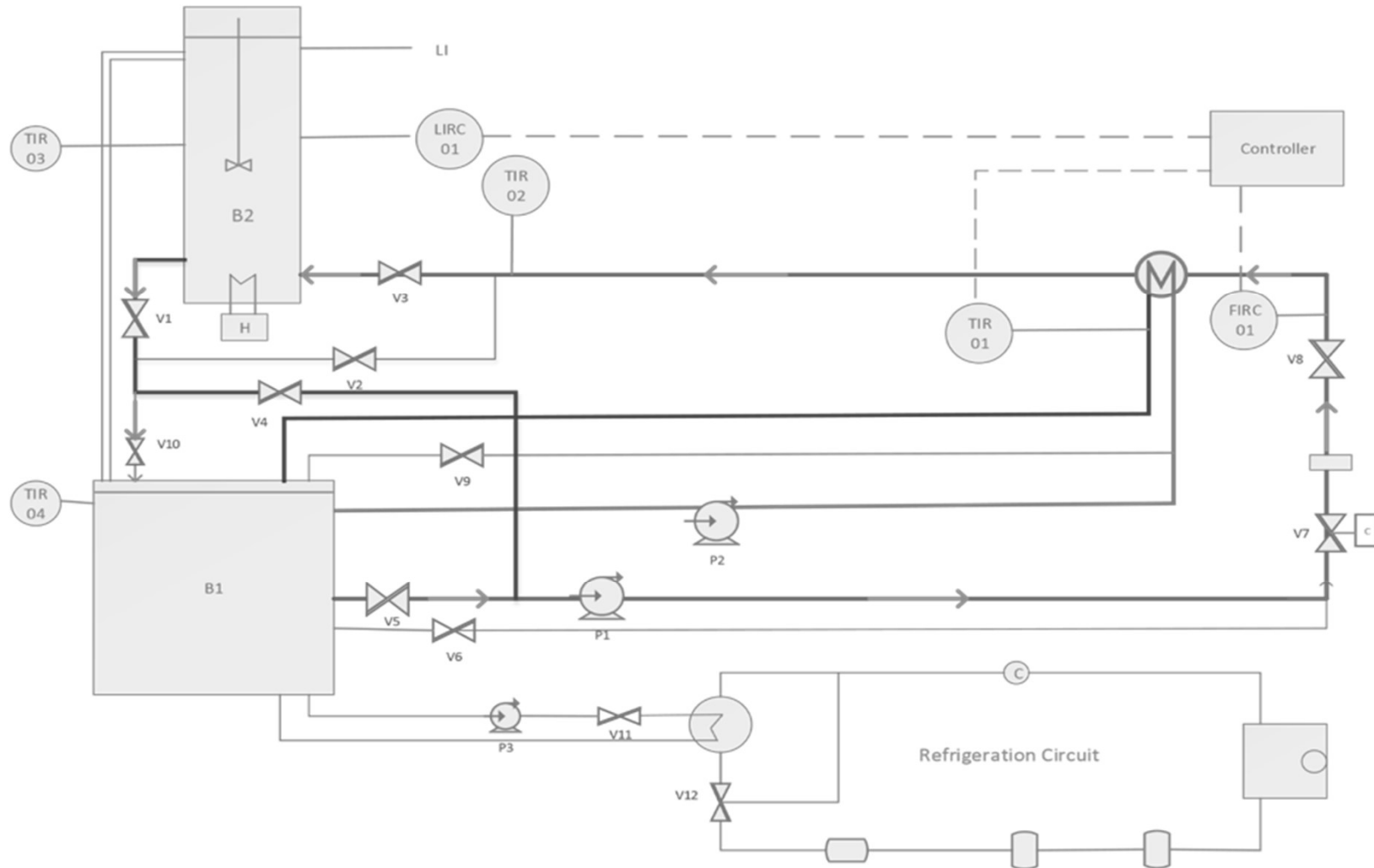
Why Copula Functions?

- To model joint probability distribution of correlated variables using dependence.
- Can capture tail dependence which the traditional joint probability distribution cannot.
- Use of rank correlation (rather than linear/Pearson correlation) such as Spearman or Kendall's Tau which depends on copula parameters.
- Easy to conduct with the given marginal distributions and correlation coefficients of variables in a flexible way

Methodology



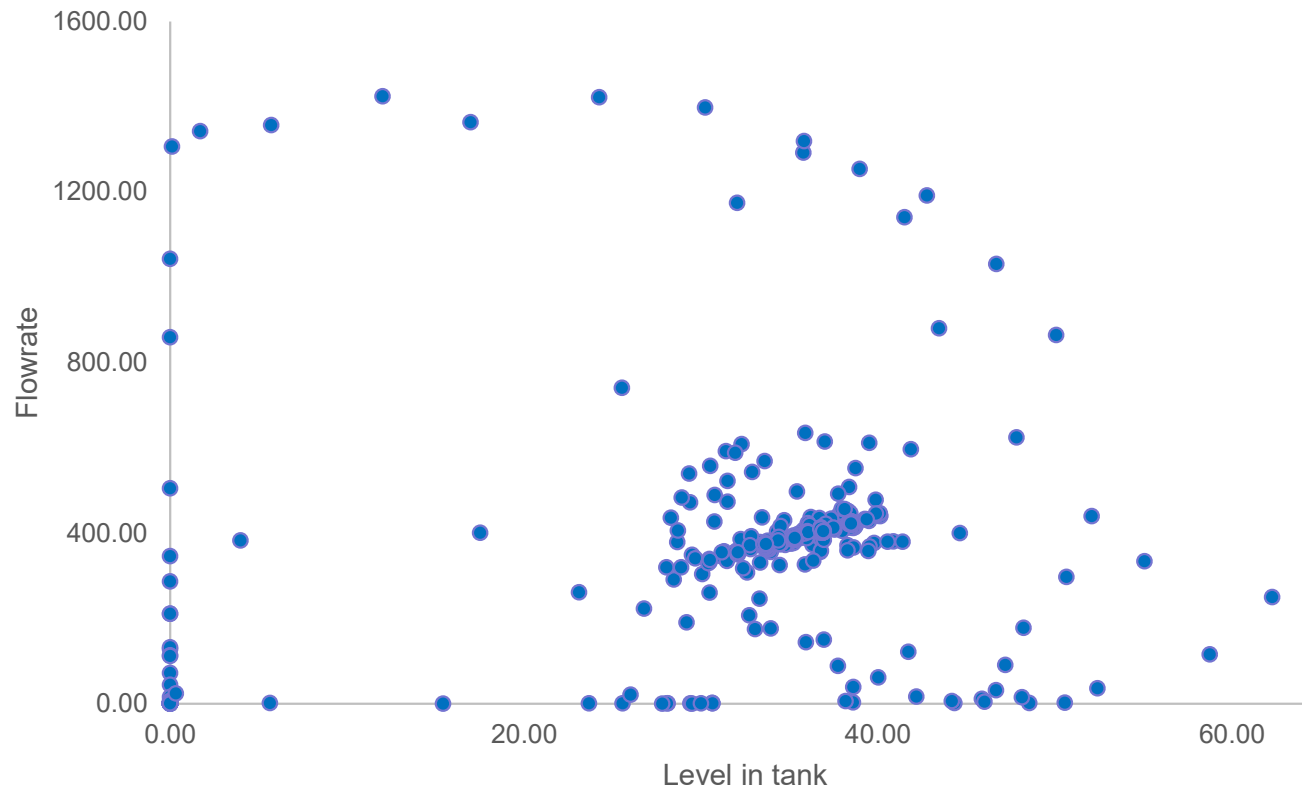
Experimental Design



Experiment Procedure

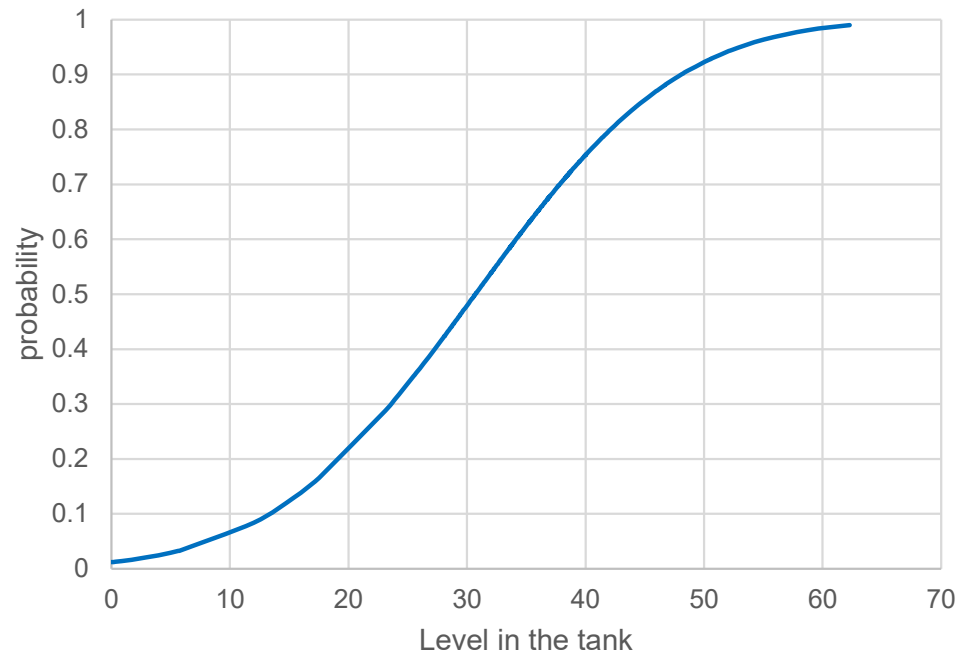
- Control: controlling level of the tank at 40%
- Fault: 'Level sensor is broken' & 'Valve V7 failed close'
- Accident Scenario: 'Dry out' in the process tank
- Duration of collecting data: 420s
- Assumed Threshold:
 - Level in the tank $<10\%$ is dry out
 - Flowrate <200 l/h is dry out

Experiment Data

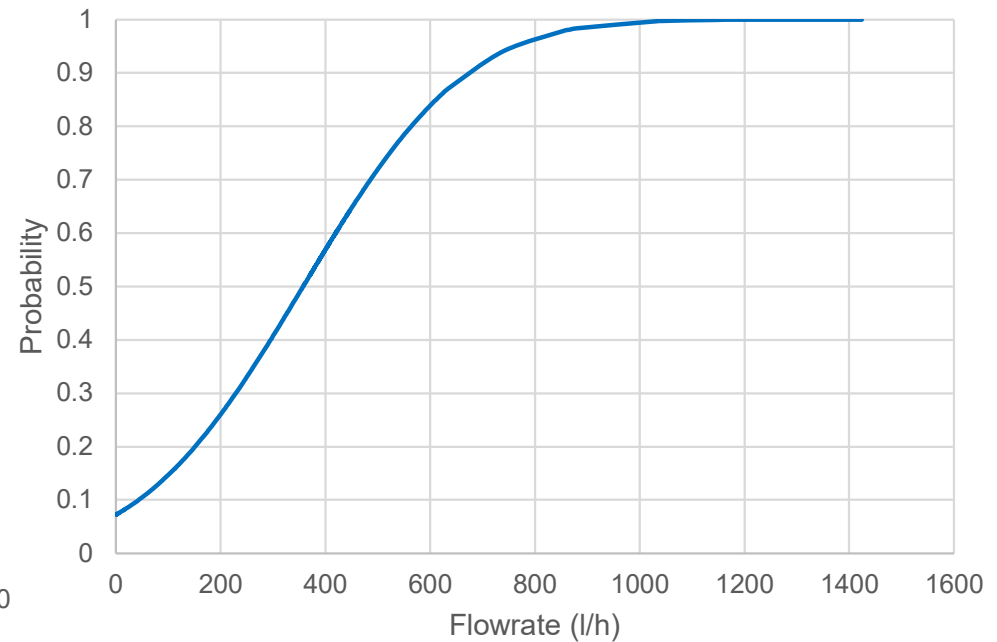


Marginal Distributions of variables

CDF of Level in the tank



CDF of Flowrate



Results

Experimental Dry out Probability	0.06
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Copula family	Kendall's Tau, $\tau =$ 0.4356	Parameter	Tail dep.	Dry out probability
Gaussian		0.63	0	0.048
Student t		0.6356, dof =4	0.40	0.049
Clayton		1.54	Lower= 0.64	0.059
Gumbel		1.77	Upper =0.52	0.043
Independent				0.013

Discussions

- ✓ Risk analysis of multivariate process system considering the variables independent is no longer valid.
- ✓ Risk analysis considering the nonlinear dependency using copula functions provides the realistic results.
- ✓ Clayton copula gives the perfect match with the experimental result
- ✓ Student T captures all four tail dependencies
- ✓ Clayton captures the lower left tail dependency
- ✓ Our experimental Level PDF is more left tailed than right tailed

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THANK YOU

QUESTIONS?