Simultaneous state and input estimation of non-linear process with unknown inputs using particle swarm optimization particle filter (PSO-PF) algorithm

Mohammad A. Khan,

CSChe 2016

Outlines

- **Motivations**
- **Tools for estimation**
- **Proposed methodology**
- **Numerical Example**
- **Conclusion**

Motivations

- **Estimation of unknown state and disturbance input from measured date.**

- **Estimated input and state can be used for process safety design.**

- **Estimated input can be used for warning system generation.**

Problem Formulation

Consider a non-linear system as follows:

$$x_{k+1} = f(x_k) + u_k + w_k$$
$$y_k = g(x_k) + v_k$$

where, x is state, u is the input and y is the measurement of a system with w_k and v_k noise .

We have to estimate both x and u simultaneously

Different approaches for estimation

	Linear	Non linear
Input known	Luenberger observer, Kalman filter	Extended Kalman Filter, Unscented Kalman Filter, Particle Filter [1]
Input unknown	Unknown input observer, Kalman based recursive Filter [2]	Maximizing a posteriori (MAP) of a estimated Gaussian posteriori [3]

Tools for Estimation

Two tools are used for this work

- **Particle filter (PF) for state estimation.**
- **Particle swarm optimization (PSO) for input estimation**

Why Particle filter

- **Kalman filter can’t show good result for non-linear system.**

- **Modified Kalman filter (EKF, UKF) does not shows good result for non-gaussian process.**

- **Particle filter weight a known distribution from current measurement and don’t require to calculate the posterior directly.**

Stages of Particle filter

MEMORIAL UNIVERSITY

There are two stages of a Particle filter:

- **Prediction -- Particles passed through state equation to obtain sample from prior**
- **Update – This stage weight the samples based on the values of measurement**

Prediction stage of PF

- **In the prediction stage N number of particle x^i is generated where i=1,2, ..N.**

- **These particles are passes through the state equation to evaluate a prior estimate of**
  $p(x_k / x_{k-1}, y_{1:k})$

Update stage of PF

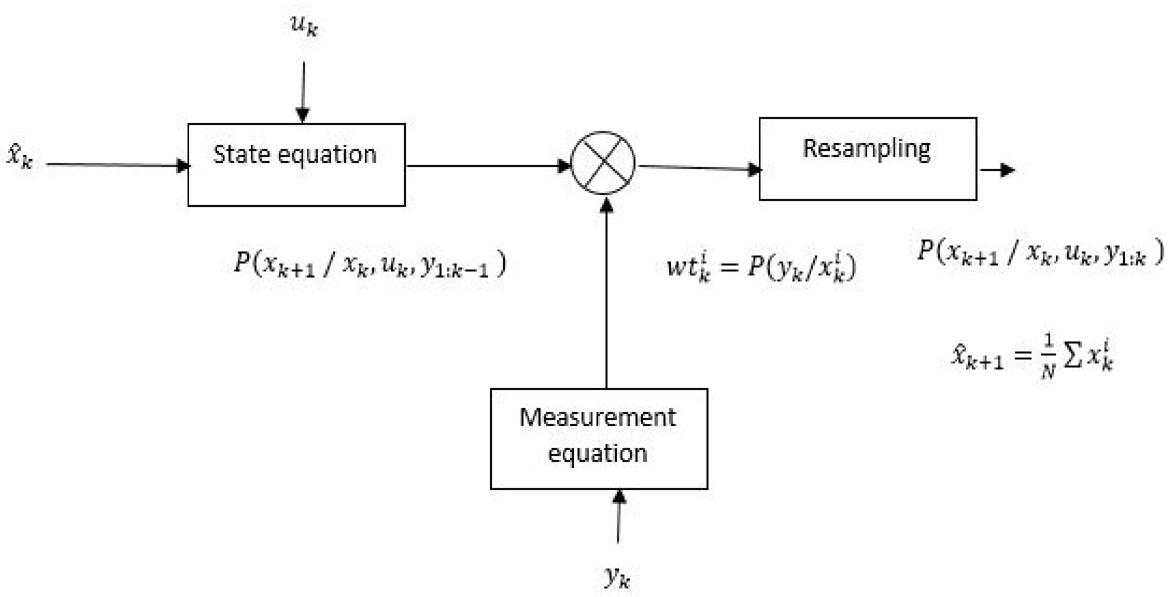
- **In this stage Bayes rules are applied to incorporate the measurement information with a recursive rule**

$$W_n \propto p(y_n/x_n) \, W_{n-1}$$

- **Particles resamples based on the intensity of weight and reset after resample.**
- **As it depends on only current value and the weight with a distribution, this estimates better for non-Gaussian.**

Implementation scheme of PF

- **Implementation scheme of PF for known input system described in [1] is shown below**



Particle Swarm optimization (PSO)

- **PSO is first proposed by Eberhart** and **Kennedy** [4].
- **PSO is motivated by the natural social behaviour of bird, ant etc.**
- **It search for a optimum solution for a function in a given space.**

PSO algorithm initialization

- **Initially population is created in a given space.**
- **Iterative methods applied on each swarm to find the optima.**
- **Optimization occurs through the learning, memory and communication among the swarms.**

Velocity and position update

- **Each swarm is passed through the function.**
- **Movement of swarm is evaluated by depending on the global best, personal best.**
- **Position of the swarm in next iteration is updated with the swarm movement.**

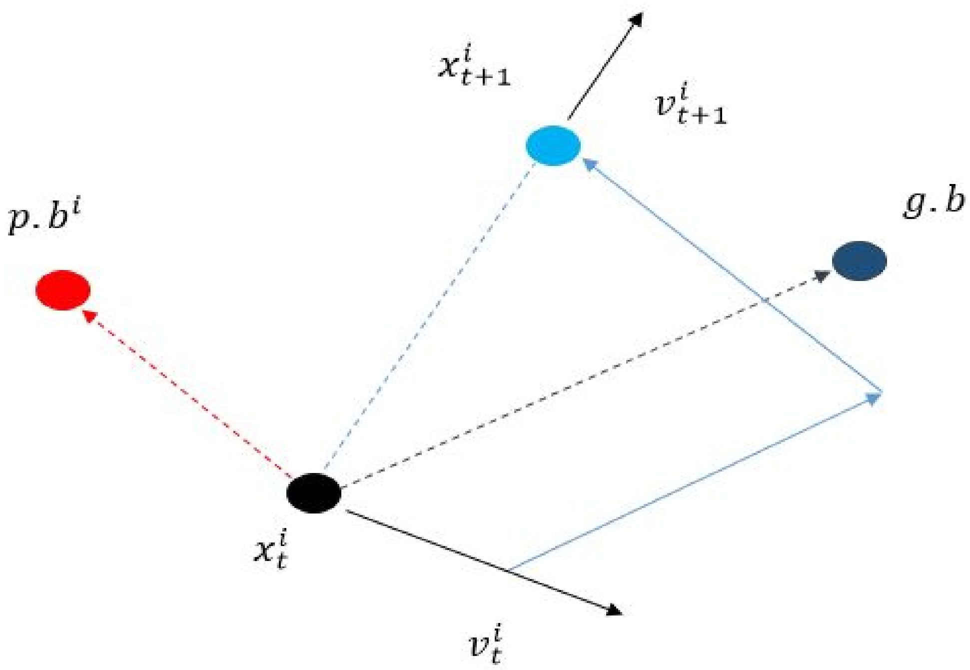
Mathematical representation

- **Swarm position and updating process is expressed with the following equations:**

- $v_{t+1}^{i} = v_t + C_1 * randn * (p.b^{i} - x_t^{i}) + C_2 * randn * (g.b - x_t^{i})$
- $x_{t+1}^{i} = x_t^{i} + v_{t+1}^{i}$

where, v_t and x_t are the velocity and position of a swarm and p.b is the personal best value for that swarm and g.b is the global value. C_1 and C_1 are fixed and chosen 2 for our case

Graphical Representation (adopted from [5])



Evaluation of global optima

- **Iterations keep continue till the difference of global optima between two iterations is lower than a specified value.**

- **For our case study we considered 1% error to be acceptable limit.**

Proposed Algorithm

Proposed algorithm has two stages

- **Input estimation stage**
- **State estimation stage**

Input Estimation Stage

In the first step input is estimated with the following steps

- x_k **from previous time step is used to calculate input free state** $\hat{x}_0$.

- **Using PSO algorithm input is evaluated minimizing,** $y - g(\hat{x}_0 + u)$**, in other word it can be express as**

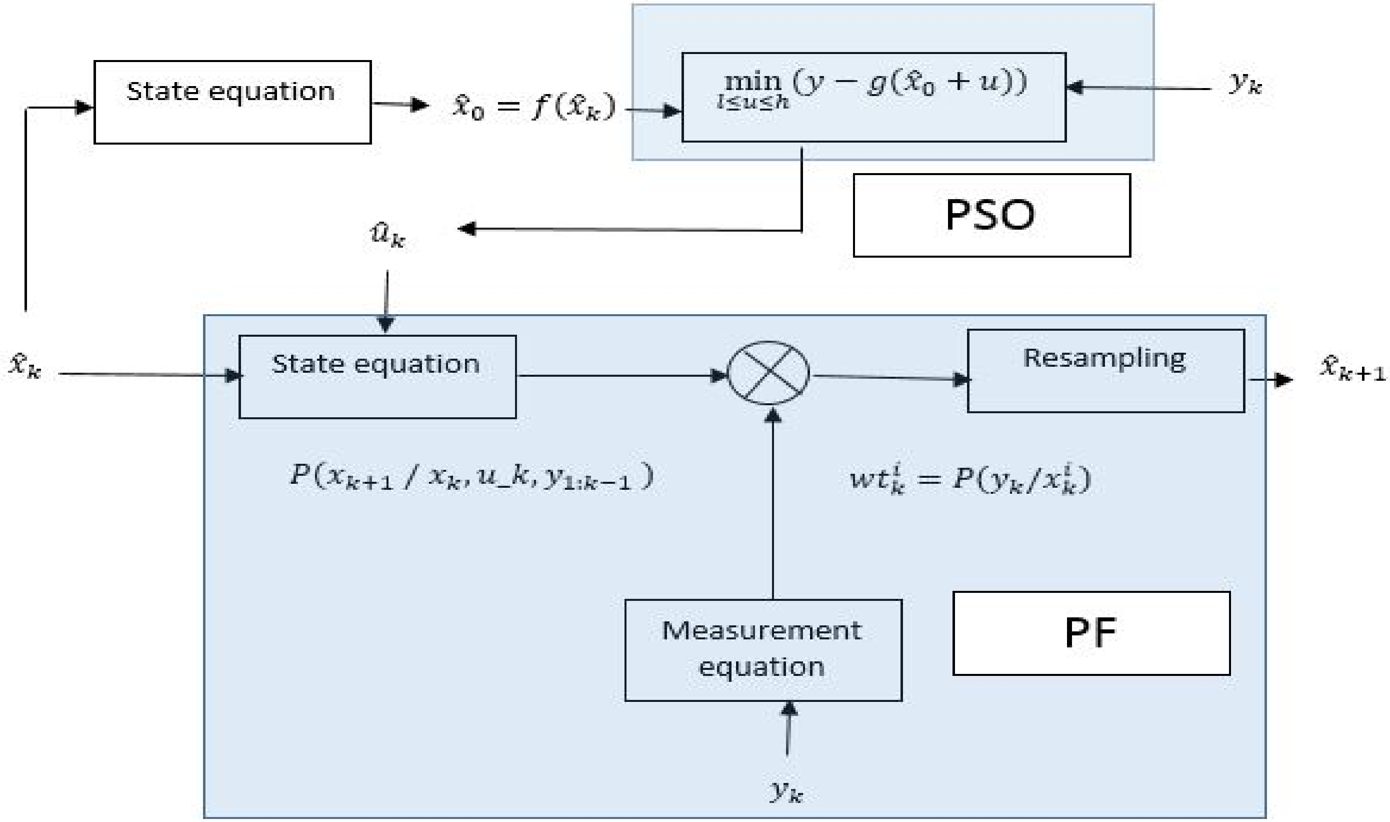
$\min_{l \leq u \leq h} (y - g(\hat{x}_0 + u))$**, to find u, where l and h is the lower and upper limit of u.**

State Estimation Stage

- **In this stage a conventional PF is employed using the state from previous time step and estimated input from input estimation stage.**

- **Using these values prior is estimated first and then weighted with the value of the measurement as was done in [1]**

Scheme of Proposed Algorithm



Numerical Example

- **We demonstrate our methodology for a simple nonlinear system as follows**

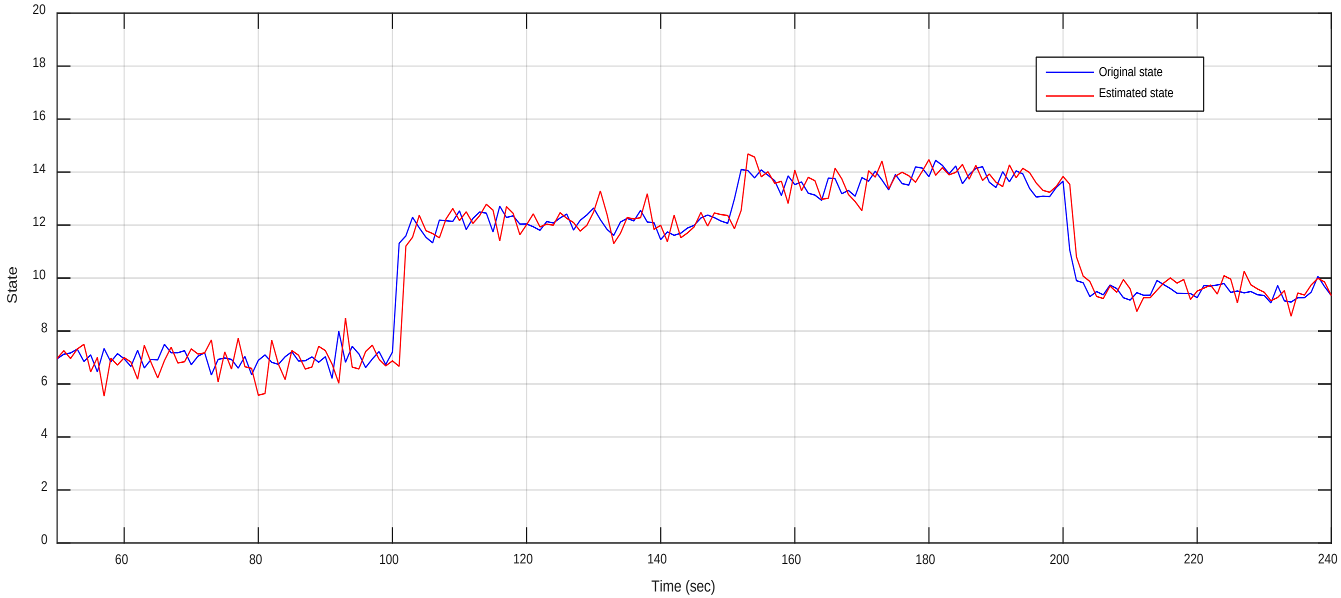
$$x_{k+1} = 0.5x_k + \frac{25x_k}{1 + x_k^2} + u_k + w_k$$

$$y_k = \frac{x_k^2}{20} + v_k$$

- **Different sizes of step inputs are combined in u.**
- **Simulation carried out with two values of noise variance (0.1 as low and 1 as high).**

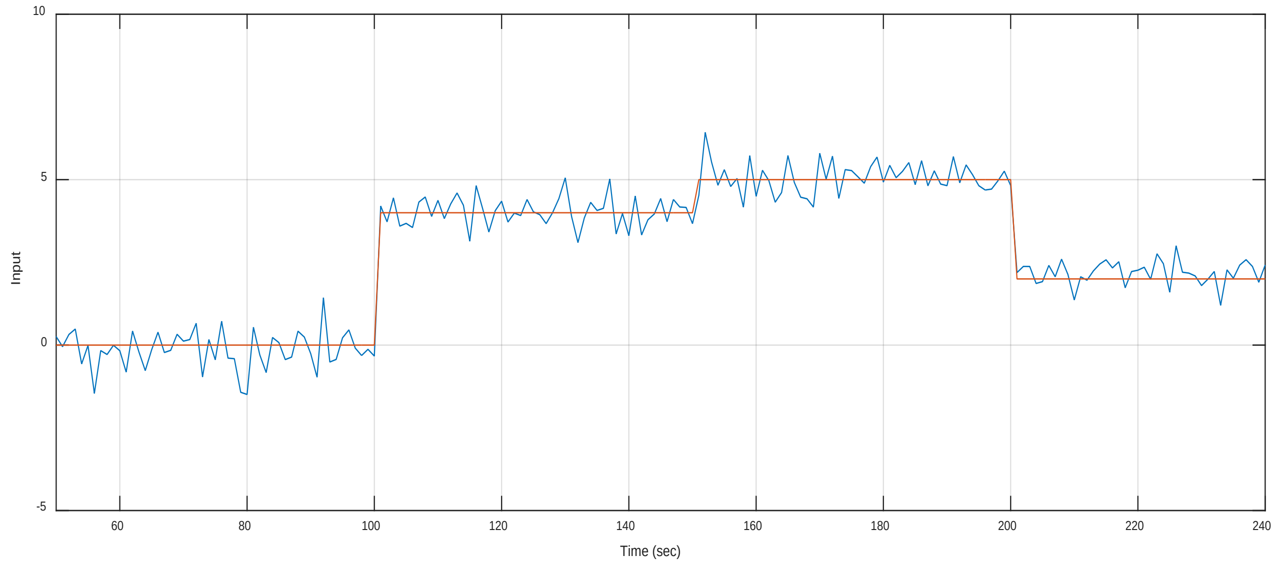
Numerical Example cont.

- **State estimation for low noise**



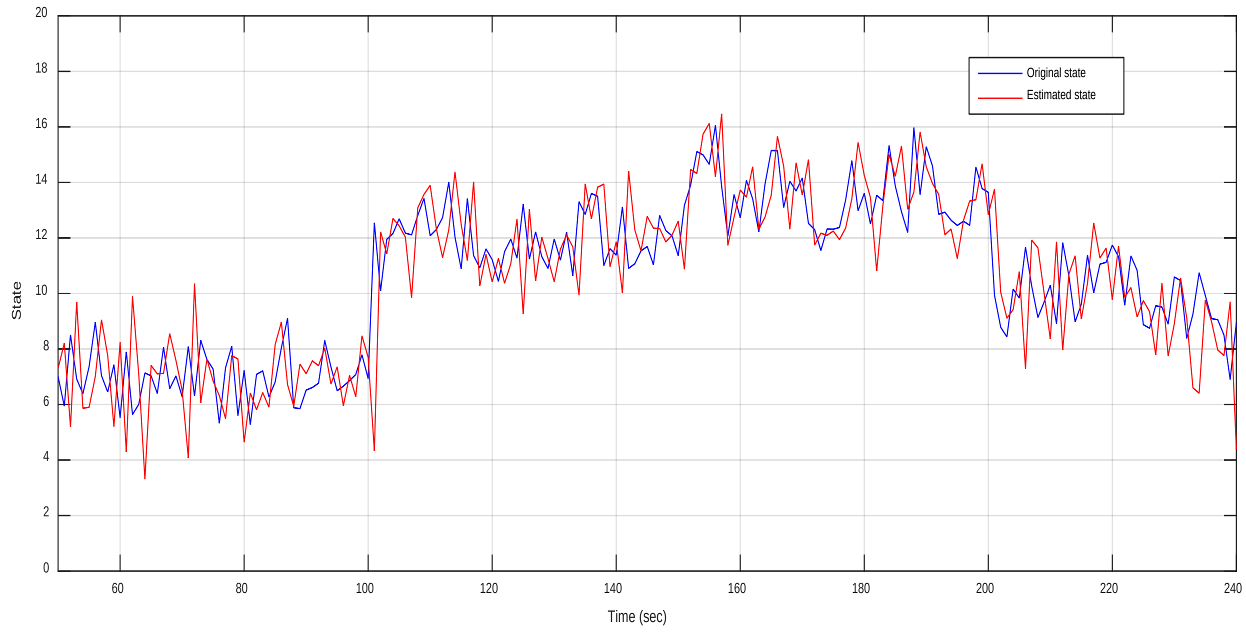
Numerical Example cont.

- **Input estimation for low noise**



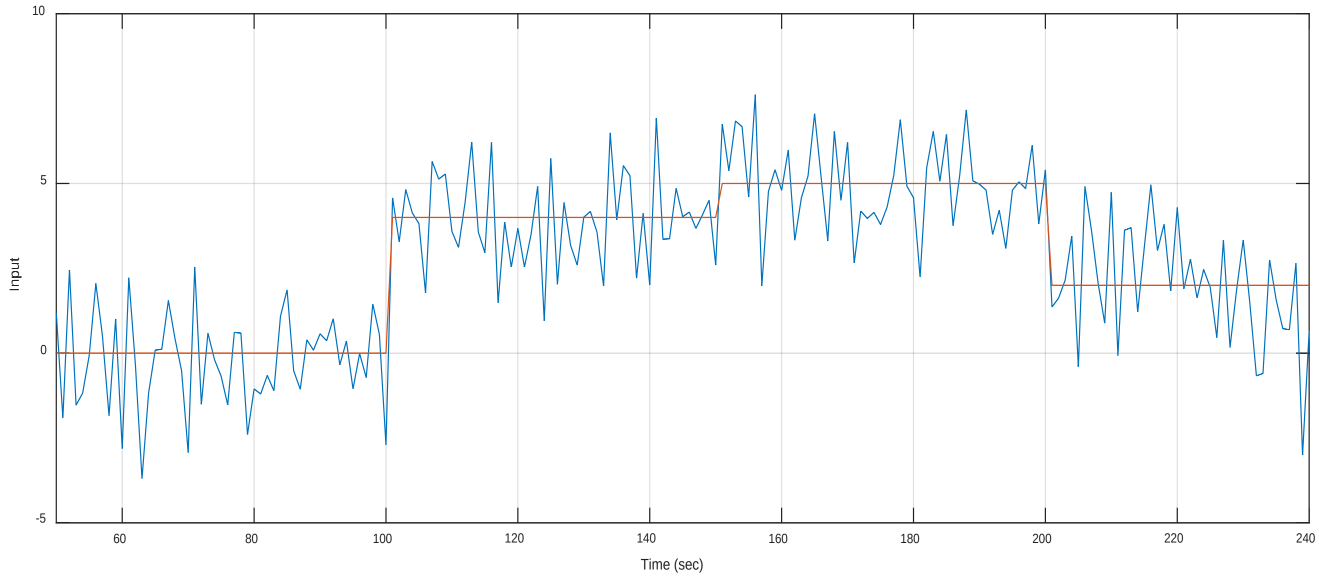
Numerical Example cont.

- **State estimation high noise**



Numerical Example cont.

- **Input estimation high noise**



Future work

- **Improvement of previously warning protocol [5] for unknown input.**
- **Used for kick detection in managed pressure drilling.**
- **Used for dynamic positioning of ship in harsh environment.**

Conclusion

- **Results show good estimation for lower noises.**
- **For higher noises estimation is not as good for small changes.**
- **Performance can be further improved with the tuning parameter of PSO in expense of higher computational complexity.**

References

- **[1]** Imtiaz, Syed A., et al. "Estimation of states of nonlinear systems using a particle filter." *Industrial Technology, 2006. ICIT 2006. IEEE International Conference on*. IEEE, 2006.

- **[2]** Gillijns, Steven, and Bart De Moor. "Unbiased minimum-variance input and state estimation for linear discrete-time systems." *Automatica* 43.1 (2007): 111-116.

- **[3]** Fang, Huazhen, and Raymond A. de Callafon. "Nonlinear simultaneous input and state estimation with application to flow field estimation." *2011 50th IEEE Conference on Decision and Control and European Control Conference*. IEEE, 2011.

References

- **[4]** Eberhart, Russ C., and James Kennedy. "A new optimizer using particle swarm theory." *Proceedings of the sixth international symposium on micro machine and human science*. Vol. 1. 1995

- **[5]** Montazeri-Gh, M., S. Jafari, and M. Nasiri. "Application of Particle Swarm Optimization in Gain Tuning of Integrated Flight and Propulsion Control."*International Journal of Aerospace Sciences* 2.3 (2013): 55-70.

- **[6]** Khan, Mohammad Aminul Islam, Syed Ahmad Imtiaz, and Faisal Khan. "Early Warning System for Chemical Processes with Time Delay and Limited Actuator Capacity." *Industrial & Engineering Chemistry Research* 53.12 (2014): 4763-4772.

Acknowledgement

Supervisors: Dr. Syed Imtiaz ,Dr. Faisal Khan

NSERC for financial support.

Thank You

Questions